COSY INFINITY Version 7

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Abstract. An overview over the features of version 7 of the code COSY IN-FINITY is given. Currently distributed to about 160 registered users, the code allows the computation and manipulation of maps of arbitrary order for arbitrary arrangements of fields. Besides the conventional analysis tools including various techniques for symplectic tracking, normal form methods, hardware and reconstructive aberration correction, and achromat design, several new features are presented. These include efficient methods to treat fringe field, a variety of methods to directly use measured field data for the computation of maps, as well as the computation and analysis of spin dynamics. Furthermore, we give references regarding the practical use of the new remainder differential algebraic methods, which among other things allow fully rigorous estimates of stability times of nonlinear repetitive motion.

INTRODUCTION

This paper provides background of the code COSY INFINITY and its language, and examples of some of the new features in COSY INFINITY [1]. These new features include the SYSCA fast fringe field methods [2–5], the computation of maps from arbitrary measured fields, the computation and analysis of spin dynamics [6,7] which is getting more important due to the increasing number of studies connected to the acceleration of polarized particles, as well as the tools used for the design of fifth order achromats [8–10], and the new remainder differential algebraic method [11–14]. Besides these features based on new techniques, there are also a variety of other new tools of a more technical nature. These are connected to standard problems of accelerator design, to interactive graphics, as well as to several improvements of existing tools. A good overview over the key features of the code can be found in the extensive demo that is part of the COSY shipment. Information about COSY can also be obtained at www.beamtheory.nscl.msu.edu/cosy.

THE CODE COSY

COSY INFINITY is a code for the simulation, analysis and design of particle optical systems. Since the first official version in 1989, a total of seven releases with an increasing number of features have been provided, and currently there are a total of about 160 registered users as shown in Figure 1. COSY is based on differential algebraic methods, which are described in detail elsewhere [15–17], and which lately have been widely used also in most of the other newly emerging codes.

For the sake of portability, the code is based on standard FORTRAN 77, which is still the most widespread language on the computers used for accelerator design and simulation. Since the code employs its own programming language which is object oriented and has the flavor of PASCAL, COSY's programmers and users are free from FORTRAN. The COSY language has very simple syntax which makes programming easy, and its elements are as follows:

```
BEGIN ;      END ;          { Begins and ends program            }
INCLUDE ;    SAVE ;         { Includes and saves compiled code   }
VARIABLE ;                  { Declares a local variable           }
```
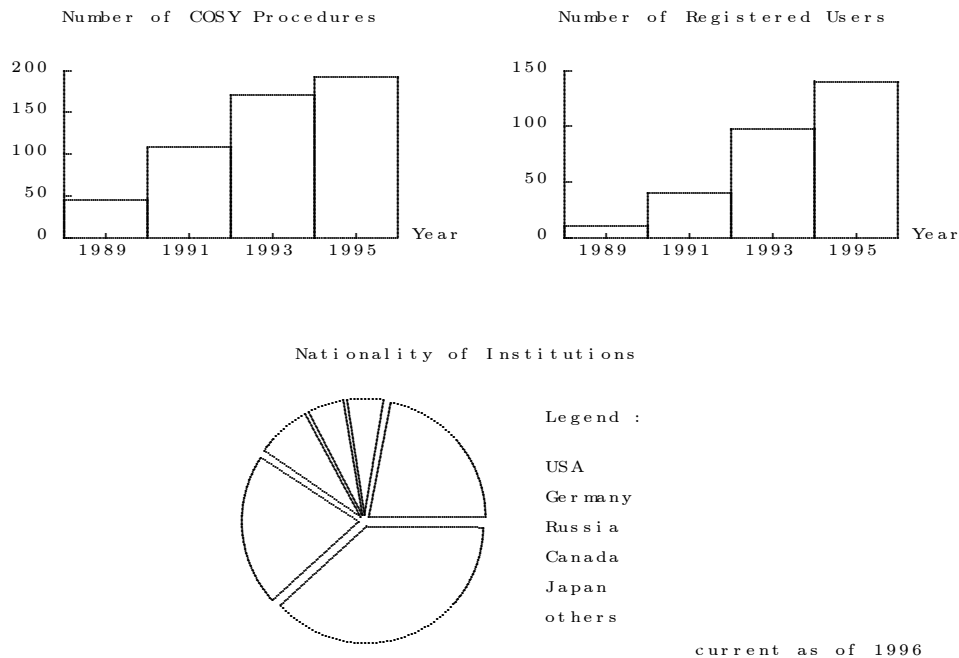

FIGURE 1. Some statistics about COSY INFINITY (picture generated with COSY's graphics environment and part of the COSY Demo).

```
PROCEDURE ;   ENDPROCEDURE ; { Declares a local procedure        }
FUNCTION ;    ENDFUNCTION ;  { Declares a local function         }
< assignments > ;           { Sets value of variable            }
< procedure calls > ;       { Calls previously defined  procedure }
IF ;          ENDIF  ;      { Executed once if argument is true  }
WHILE ;       ENDWHILE ;    { Executed while argument is true    }
LOOP ;        ENDLOOP ;     { Stepping argument                 }
FIT ;         ENDFIT ;      { varying arguments to fit conditions }
```

Except for the last one, the flow control statements are rather standard. The unique FIT block is used for efficient utilization of COSY's various optimizers. The ENDFIT statement contains the number of the optimizer to be used and the objective quantities, as well as the tolerance and the maximum number of iterations allowed. The statements in the block are executed over and over again, where for each pass the optimization algorithm changes the values of the variables listed in the FIT statement and attempts to minimize the objective quantity.

The compiler for this language comprises about 6,000 of the approximately 25,000 lines of COSY's FORTRAN code. It has a completely rigorous syntax and error analysis and is comparable in speed to other compilers. The compiled code is stored in a binary file, which can either be saved for inclusion in later code or executed directly. The object oriented features of the code allow a direct use of the differential algebraic operations contained in the 16,000 line DA package.

THE FAST FRINGE FIELD CALCULATION

COSY has several options to take account of the fringe field. One is to compute the exact fringe field through DA integration of the equations of motion. In this case, the accuracy is limited only by the accuracy of the numerical integrator, which is adjusted via automatic step size control. While this method provides a very detailed fringe-field calculation, the computational expense becomes quite high. One of the other approaches to compute fringe-field in COSY is the SYSCA method. It is based on a combination of geometric scaling in TRANSPORT coordinates and symplectic rigidity scaling [2–5].

It uses parameter dependent symplectic representations of fringe-field maps stored on files to approximate the fringe field via symplectic scaling. This method computes fringe fields with a very high accuracy, yet its computational expense is two or three orders of magnitude less than direct integration and in the same terrain as that of main field only calculation. While the reference file can be produced by a user in COSY according to the detailed shape of the fringe fields at hand, one standard reference file comes as part of the COSY shipment. The SYSCA fringe-field mode is especially helpful in the final design stages of a realistic system after approximate parameters of the

elements have been obtained by neglecting fringe fields or with other rough fringe-field calculation.

THE MEASURED FIELD DATA ELEMENT

One of new elements in COSY INFINITY, the measured field data element, allows the computation of the map of any magnetic field supplied by midplane measured data. The measured data has to be supplied at equi-distant grid points in cartesian coordinates.

The necessary interpolation method has to guarantee the differentiability to fit with the differential algebraic computation. Because of this, the evaluation of the field strength in the element in COSY is done by Gaussian interpolation, which is a powerful special case of a wavelet transform:

$$B_y(x, z) = \sum_{i_x, i_z} \mathrm{BY}(i_x, i_z) \frac{1}{\pi \mathrm{S}^2} \exp\left[-\frac{(x - x_{i_x})^2}{\triangle x^2 \mathrm{S}^2} - \frac{(z - z_{i_z})^2}{\triangle z^2 \mathrm{S}^2} \right],$$

where $\mathrm{BY}(i_x, i_z)$ are the supplied measured data. Since the gaussian function falls off quickly, the time consuming summation over all the gaussians can be replaced by the summation of only the neighboring gaussians, which is in the vein of other wavelet transforms and greatly improves efficiency.

The method is used extensively in the simulation of the S800 Spectrograph at NSCL/MSU [18], construction of which has just been completed, as well as the various spectrographs at CEBAF.

SPIN DYNAMICS

In version 7, features to analyze spin motion have been added in COSY [6,7]. The classical equation of motion for spin has the form

$$\frac{d\vec{s}}{dt} = \vec{w} \times \vec{s};$$

for details, see [7]. The solution is a linear orthogonal transformation depending on orbital variables, thus

$$\vec{s}_f = \hat{A}(\vec{z}) \cdot \vec{s}_i, \quad \text{where} \quad \hat{A}(\vec{z}) \in SO(3).$$

The motion of a particle with spin can be described as a nine dimensional motion, neglecting spin-orbit coupling, as

$$\left(\begin{array}{c} \vec{z} \\ \vec{s} \end{array} \right)' = \vec{F}(\vec{z}, \vec{s}, s) = \left(\begin{array}{c} \vec{f}(\vec{z}, s) \\ \hat{W}(\vec{z}, s) \cdot \vec{s} \end{array} \right)$$

$$\begin{pmatrix} \vec{z}_f \\ \vec{s}_f \end{pmatrix} = \vec{M}(\vec{z}_i, \vec{s}_i, s) = \begin{pmatrix} \mathcal{M}(\vec{z}_i, s) \\ \hat{A}(\vec{z}_i, s) \cdot \vec{s} \end{pmatrix} \quad .$$

To reduce dimensionality and utilize linearity, it is advantageous to set up the equation of motion for $\hat{A}$. Insertion yields the equation of motion for the 3×3 spin matrix depending on only the six orbital variables:

$$\hat{A}'(\vec{z}, s) = \hat{W}(\vec{z}, s) \cdot \hat{A}(\vec{z}, s).$$

The COSY spin features have been used extensively at DESY and a variety of other places.

OTHER FEATURES

A recent application of COSY was the design of higher order achromats, and recently some extensive work connected to fifth order achromats has been done [8–10].

The new remainder differential algebraic (RDA) method combines the rigor of interval computations and a reduction of blow-up due to its use of Taylor polynomials. For the purposes of beam physics, it allows the determination of rigorous bounds for the remainder term of Taylor maps, and combined with methods to determine approximate invariants of the motion, it can be used for guaranteed estimates of stability times in circular accelerators [11–14].

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